Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

MATHEMATICS

(M_7 : Partial Differential Equations & Calculus of Variation)

Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the integral curves of the equations

$$\frac{dx}{x+z}=\frac{dy}{y}=\frac{dz}{z+y^2}.$$ 6

(B) Verify that the equation

$$2xz\,dx - 2yz\,dy - (x^2 - y^2)(z - 1)\,dz = 0$$

is integrable and hence solve it. 6

OR

(C) Verify that the equation

$$z(z + y^2)\,dx + z(z + x^2)\,dy - xy(x + y)\,dz = 0$$

is integrable and find its solution. 6

(D) Eliminate the arbitrary function from the equation

$f(x^2 + y^2 + z^2, z^2 - 2xy) = 0.$ 6

UNIT—II

2. (A) Find the general solution of the differential equation

$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z.$$ 6

(B) Find the integral surface of the linear partial differential equation.

$2y(z - 3)p + (2x - z)q = y(zx - 3)$

which passes through the circle $Z = 0,\ x^2 + y^2 = 2x$. 6

OR

(C) Show that the equations xp = yq, z(xp + yq) = 2xy are compatible and solve them. 6

(D) Solve the equation $p^2x + q^2y = z$ by using Jaccobi's method. 6

UNIT—III

3. (A) Solve

$$(D^2 - 2DD' + D'^2)\, z = e^{x + 2y} + x^3.$$ 6

(B) Solve

$$(D^2 - 2DD' + D'^2)\, z = 2 \cos y - x \sin y$$

where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$. 6

OR

(C) Solve

$$(D^2 - DD' + D' - 1)\; z = \cos (x + 2y) + e^y$$

where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$. 6

(D) Solve

$$y^2 \frac{\partial^2 z}{\partial y^2} - y \frac{\partial z}{\partial y} = xy^2,$$ by using

$x = e^u$, $y = e^v$. 6

UNIT—IV

4. (A) Test for the extremum of the functional

$$I[y(x)] = \int_0^1 [(y')^2 + (y') + 1]\, dx,\ y(0) = 1$$

y(1) = 2 and show that the extremal is a straight line y = x + 1. 6

(B) Find the curves on which the functional $I[y(x)] = \int_1^2 \frac{x^3}{(y')^2} dx$ with y(1) = 0 and y(2) = 3 can be extremized. 6

OR

(C) In the problem of determining extremals of the functional

$$I[y(x)] = \int_0^{\log 2} [e^{-x}(y')^2 - e^x y^2]\,dx$$ 6

verify the invariance of Euler's equation under the coordinate transformation.

(D) Write Euler-Ostrogradsky's equation for the functional

$$I[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy\,.$$ 6

QUESTION—V

5. (A) Solve the equation

$a^2y^2z^2\,dx + b^2z^2x^2\,dy + c^2x^2y^2\,dz = 0$, by separating variables. 1½

(B) Eliminate the constants a and b from the equation

$Z = (x + a)(y + b)$. 1½

(C) Solve

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z\,.$$ 1½

(D) Write the charpit's auxiliary equations for

$xp + 3yq = 2(z - x^2q^2)$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. 1½

(E) Solve

$$(D - D')\,z = 0, \text{ where } D = \frac{\partial}{\partial x},\; D' = \frac{\partial}{\partial y}\,.$$ 1½

(F) Find PI of $\left(D^2 - 3DD' + 2D'^2\right)$ $z = \sin(x - 2y)$. 1½

(G) Find the distance of order zero between the functions $y = x^2$ and $y = x$ on the interval [0, 1]. 1½

(H) If the function F depends on y alone i.e., if $F = F(y)$, then prove that Euler's equation assumes the form $\frac{\partial F}{\partial y} = 0$. 1½